Projectile Motion

Objective

The objectives of this experiment are for you to:

1. Develop confidence in your ability to use the equations of motion to predict the results of an experiment.
2. Gain confidence in the equations of projectile motion and your ability to use them.
3. Further develop an appreciation of analysis of errors in physical measurements.

Materials

1. 1-meter stick (x2)
2. Tape measure
3. Ballistic pendulum machine
4. Plywood (wall protection)
5. Projectile (metal ball)
6. White paper / masking tape

Introduction

Projectile motion is a special type of motion in two dimensions. Projectile motion is two-dimensional motion that takes place in a uniform gravitational field and whose motion is influenced only by that gravitational field -- i.e., all sources of resistance are negligible. Even more specifically, the term is typically applied when the motion takes place near the surface of the earth, where Earth's gravitational field is approximately constant and has the value 9.8 m/s^2. Under these circumstances the acceleration of the object is constant (and occurs only in the vertical direction), therefore the kinematic equations apply.[1] See any Physics textbook for their derivation.

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \tag{1}$$

$$v_x = v_{x0} + a_x t \tag{2}$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0) \tag{3}$$

$$x = x_0 + \bar{v}t = x_0 + \frac{1}{2}(v_{x0} + v_x) \tag{4}$$

Note that x is final position, x_0 is initial position, v_x is final velocity, v_{x0} is initial velocity, t is time, a_x is acceleration in the x-direction. Note that these equations may be rewritten for the y direction (or any direction) by substituting y for x.

Procedure

To accomplish the objectives above, you will first determine the launch speed of a projectile by making measurements of its horizontal and vertical displacements when it is shot horizontally. Then you will use this initial speed to predict the landing point of the projectile when it is shot at an angle θ above

[1] For some reason some textbooks/references do not include Equation (4).

horizontal. You will make use of the projectile launcher (but not the pendulum arm) of a ballistic pendulum device, as follows.

1. With the projectile launcher oriented horizontally -- i.e., level with the ground such that the inclinometer (a.k.a., “clinometer") reads 0 -- insert the projectile into the barrel of the launcher. Use the plastic ramrod to shove the projectile back until the yellow indicator inside barrel indicates the ”LONG RANGE" position. The launcher is now cocked and ready to fire the projectile. Lift up on the yellow string to launch the projectile. Make appropriate distance measurements to allow you to determine the speed with which the ball was launched. Choose +y to be upward and the origin at the launch point; then the vertical motion is described by Equation (1), rewritten in terms of y. Solve this equation for t as a function of y. Using this t you may obtain the initial velocity from Equation (1), written in terms of x. (Hint: What value does a_x have?) Perform the experiment five times; compute the five launch speeds and their average.
2. Calculate s_v, the standard deviation of v_0. How may we interpret this standard deviation? (See Addendum.)
3. Notify your instructor that you have determined the launch speed. He/She will then set the launcher at an arbitrary angle θ above horizontal. Use the clinometer (a.k.a., “inclinometer") to measure this angle, and calculate where the ball should strike the floor. With +y upward and the origin at the starting point, write Equation (1) for y. (Hints: What is y_0? What is a_y? What is v_{y0}?) Will y be positive or negative? Solve for t. Calculate x from Equation (1) (written in terms of x). (Hints: What is x_0? What is a_x? What is v_{x0}?)
4. Use chalk to mark on the floor your predicted spot. Then calculate and mark the two positions where the ball would hit if its velocity were $v_0 \pm s_v$. Draw parallel lines (perpendicular to the plane of the ball's motion) through all three points that you have marked.
5. Launch the projectile while the instructor observes. Does it hit your middle line? If it hits between your outside lines, you were successful. If it misses by more than twice that far from your predicted distance (what is the probability of this?), you definitely need to check your analysis and repeat the experiment using a different angle.

The lab worksheet will require you to include your data, your best estimate of v_0, your predictions, and your results. Should the ball always land between the outside lines? Explain. Were the objectives of the experiment achieved? Justify your answer.

Addendum: Statistical Analysis of Data

Mean

The mean of a group of values is simply the sum of the values divided by the number of values in the group. (Another name for the mean is the arithmetic average.) Mathematically expressed, the mean of a group of values x_1, x_2, x_3, . . . , x_N is given by

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Deviation

The deviation is the difference between an individual value and the mean of the group.

$$\delta x_i = x_i - \bar{x}$$

If you sum all the deviations of any group you obtain zero. That is

$$\sum_{i=1}^{N} \delta x_i = \sum_{i=1}^{N} (x_i - \bar{x})^2$$

You should either prove this or take a set of numbers and verify the statement. Obviously the sum of the deviations does not tell how the values scatter about the mean.

Standard Deviation

A way that the deviations can give information concerning the grouping of the values about the mean is by the squares of the deviations. The standard deviation, s, is defined to be

$$s = \left[\frac{1}{N-1}\sum_{i=1}^{N} (x_i - \bar{x})^2\right]^{\frac{1}{2}}$$

Certainly a relatively small value of s denotes a close grouping about the mean, whereas a relatively large value of s denotes a wide scattering about the mean. If you assume that your values have a normal distribution (see Figure 1) (bell-curve), meaning that errors are completely random, then the following statements are true.

1. 68.26% of the values are within one standard deviation s of the mean µ. (Note that s and µ are the notations for standard deviation and mean for normal distributions, respectively.)
2. 95.44% of the values are within 2s of µ.
3. 99.74% of the values are within 3s of µ.

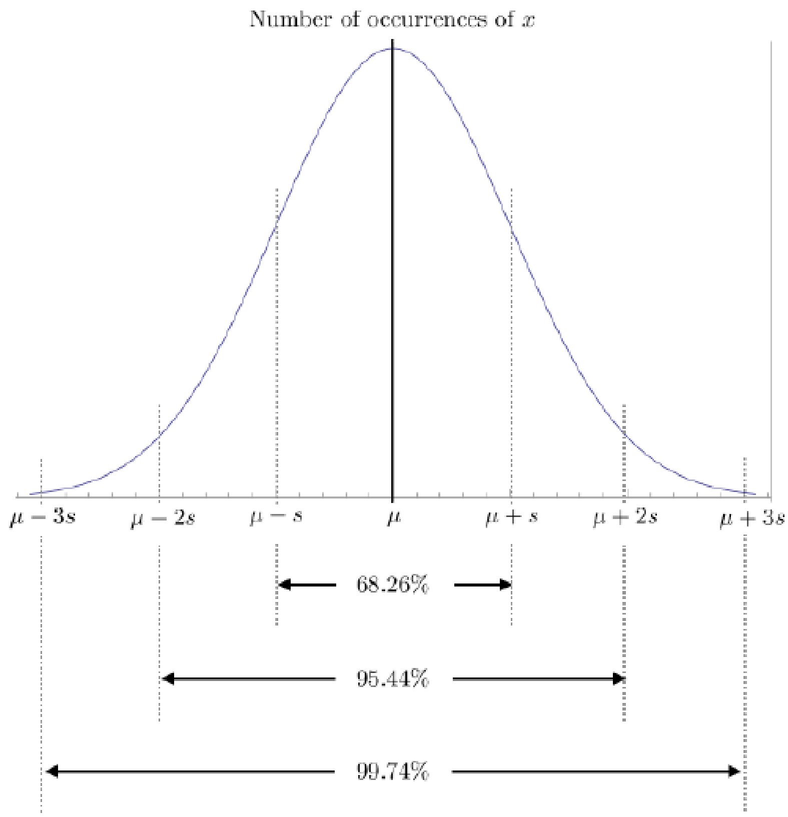

Figure 1: *s and μ are the notations for standard deviation and mean for normal distributions respectively. 68.26% of the values are within 1s of μ. 95.44% of the values are within 2s of μ. 99.74% of the values are within 3s of μ.*

We can see from these statements that standard deviation gives us an idea of how much to expect one trial of the experiment to differ from the mean of several trials. Let us note, however, that while the error in the mean becomes smaller for a larger number of trials, the standard deviation doesn't change; therefore standard deviation is not a measure of error in the mean.