Ballistic Pendulum

Objective

The purpose of this experiment[1] is to determine the muzzle velocity of a projectile launcher, using the conservation of momentum and the conservation of energy (where applicable).

Materials

1. 1-meter stick
2. 30-cm ruler
3. Ballistic pendulum machine
4. Projectile (metal ball)
5. Triple-beam balance

Introduction

The ballistic pendulum is a classic method for determining the velocity of a projectile; originally, it was used to determine the muzzle velocity of firearms. It is also a good demonstration of many basic principles of physics.

In ballistic pendulum experiments a projectile (in our case a ball) is fired into a stationary, freely hanging pendulum, which swings up by a measured amount after being impacted by the projectile. From that measured angle and from the length of the pendulum, you can determine the height reached by the center of mass of the pendulum; then you can calculate its gravitational potential energy. We know that mechanical energy (ME) is conserved, so we know that the gravitational potential energy at the maximum height must be equal to the kinetic energy of the pendulum at the bottom of the swing, just after the ball collides with the pendulum. In symbolic terms,

$$ME_i = ME_f \tag{1}$$

Where ME = PE + KE. So,

$$\cancel{PE_i} + KE_i = PE_f + \cancel{KE_f} \tag{2}$$

You cannot, however, equate the kinetic energy of the pendulum after the collision with the kinetic energy of the ball before the swing, because the collision between ball and pendulum is inelastic, and kinetic energy is not conserved in inelastic collisions. Momentum, however, is conserved in all forms of collisions, so you know that the momentum of the ball before the collision is equal to the momentum of the pendulum after the collision. Once you know the momentum of the ball and the mass of the ball, you can determine the initial velocity, vb, of the ball.

Please note that the subscript "cm" used in the following equations stands for "center of

[1] Some of the text of this manual was taken from the Pasco Ballistic Pendulum/Projectile Launcher Instruction Manual.

mass".

Procedure

Theory

The apparatus used in this experiment consists essentially of a spring-type gun that shoots a metal ball and a pendulum that catches the ball. The collision causes the pendulum to recoil. An angle indicator moves with the pendulum and stops at the highest point, marking the maximum angle of the pendulum's motion.

After the pendulum comes to a stop at the highest point, the kinetic energy is zero. (Why is it zero?) So all of the mechanical energy is in the form of potential energy. In this case, the only potential energy is gravitational potential energy. Thus, at the highest point the total energy in the system is simply the gravitational potential energy,

$$PE_g = (M + m)g\Delta h \quad (3)$$

where Δh is the change in height of the ball and pendulum. Calculate Δh. Using Figure 1, consider that $x = R_{cm} - \Delta h$. Find an expression involving only Δh, Rcm, and θ. Solve that expression for Δh. Substitute that expression into Equation (3).

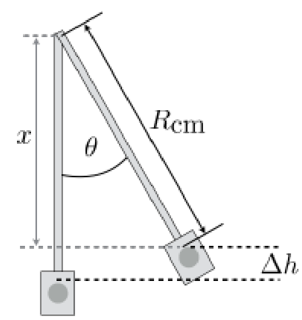

Figure 1

Immediately after the collision but before the pendulum rises, the potential energy can be set to zero. (Why does this work?) So, the total energy in the system is simply the kinetic energy of the (now moving) pendulum arm, which includes the ball; so,

$$KE = \frac{1}{2}(M + m)V^2 \quad (4)$$

where M and m are the masses of the pendulum and ball, respectively, and V is the speed with which the pendulum moves immediately after the collision.

Now apply the conservation of momentum before and after the collision. Let m be the mass of the ball, M be the mass of the pendulum arm, v_b be the speed of the projectile before the collision, and V be the speed of the pendulum and ball together after the impact. Now combine your expression from the conservation of energy with your expression from the conservation of momentum. You should ultimately find an expression for the speed of the ball before the collision -- i.e., the muzzle velocity of the projectile -- in terms of M, m, g, R_{cm}, and θ. Now, to find a numerical value for v_b, you know which measurements you need to make and how to use them!

Experiment

Gently remove the pendulum arm from the apparatus. Measure the mass of the pendulum arm. Measure the mass of the ball and pendulum together (with the ball inside the pendulum arm). Calculate the mass of the ball.

Set a meter stick on its side on the lab table. With the ball in the pendulum arm, find the center of mass by balancing it on the narrow edge of the meter stick. Measure the distance between the pivot point of the pendulum arm and the center of mass; this is R_{cm}. Before measuring R_{cm}, ensure that the cylindrical weights are snug against the pendulum casing. Why is it important to measure R_{cm} with the ball in the pendulum?

Reinstall the pendulum arm onto the stand. Load the ball into the barrel of the launcher. Gently lift the pendulum arm out of the way, and use the ramrod to compress the spring and ball to LONG RANGE.

Reset the angle indicator to zero. Note that it may not be exactly on zero. If it's not exactly on zero, then note the initial reading; you'll need to correct your measurements by that amount (aka “angle offset”).

Make sure everyone and everything is clear and not in danger of being struck by the ball (should it miss) nor the pendulum arm as it moves. Use the string to pull up the trigger to launch the ball into the pendulum. Read the maximum angle from the angle indicator. Repeat the process 10 times. For each measured value of the angle calculate the muzzle velocity. Find the average value of the muzzle velocity of the projectile.

Comparison with Kinematics

Now you will also determine the initial speed of the ball from its projectile motion (an option not available for firearms for obvious reasons). You will determine the initial velocity in precisely the same way you did earlier in the semester in the “Projectile Motion" lab. Carefully remove the pendulum arm and set it aside, and fire the ball horizontally from the tabletop, allowing it to hit the floor. The measured vertical displacement and known acceleration due to gravity should allow you to compute the time that the ball is in flight. The time and the measured horizontal displacement should allow you to compute the speed with which the ball is fired.

This will also be the speed of the ball just before its collision with the pendulum. How does this velocity compare with that determined using the ballistic pendulum?